It has been my great fortune to have had many chances to present different parts of this work to various kinds of audience. I received very useful comments and encouragement from participants in the Indiana University Logic Group seminar, in a workshop at Logic in Computer Science, in the Thinking with Diagrams workshop (University of Wales), in the University of Virginia Philosophy Department colloquium, and at the First International Conference on the Theory and Application of Diagrams (University of Edinburgh). This work especially benefited from discussions with Gerry Allwein, Peter Cheng, George Furnas, John Howse, Robert Lindsay, Patrick Scotto di Luzio, Bernd Meyer, Arthur Miller, Fernando Molina, Jesse Norman, Peter Ochs, Nik Swoboda, John Taylor, and Dirk Van Gucht.

Jim Cargile’s and Paul Humphreys’ helpful comments and strong support for my work were absolutely crucial at the final stage of the work while I was spending a sabbatical at the University of Virginia. I truly appreciate the careful reading and extremely valuable suggestions by Keith Stenning, Len Olsen, Richard Shedenhelm, and anonymous reviewers of the MIT Press. Carolyn Anderson, Michael Sims, and Alan Thwaits at the MIT Press have been a great help.

I acknowledge financial support from an NEH Summer Fellowship, a Faculty Research Award from the University of Notre Dame, and financial assistance from the Philosophy Department of the University of Notre Dame. I also greatly appreciate the kind assistance I received from Houghton Library, Harvard University, during numerous visits I paid while working on the manuscript.
Throughout every stage of the work, Jon Barwise’s, John Etchemendy’s, Jaegwon Kim’s, and John Perry’s encouragement and moral support have been very important to me. As always, my special thanks go to my mother, who has been praying for me in Korea, and to my husband, Henry Smith, whose love, help, and support have been beyond description.
1

Introduction

A fresh approach to reasoning and representation has been emerging at the crossroads of the philosophy of mind, cognitive science, logic, and computer science.\textsuperscript{1} Our ordinary reasoning typically involves information obtained through more than one medium—sentences, diagrams, smells, sounds, and so on. Recognizing the actual practice of this multi-modal reasoning, researchers have started focusing on multi-modal, or heterogeneous, representation systems, which employ both symbolic and diagrammatic elements. This is a clear departure from the major direction taken by logicians and mathematicians since the development of modern logic: For more than a century, symbolic representation systems have been the exclusive subject for formal logic.

This book analyzes a well-known, but much-criticized, non-symbolic representation system, Peirce’s system of Existential Graphs (henceforth, ‘EG’), and presents a new approach to EG based on the discovery of its unique nature and on the reconstruction of Peirce’s theory of representation. I will explore specific differences between symbolic and diagrammatic systems both in reading-algorithms and in the formulation of inference rules, and I will draw out the implications of these results for several long-standing debates in various disciplines that study multi-modal systems. Before locating this work within the overall picture of research into heterogeneous reasoning, it will be useful to re-examine two related major assumptions which underlie the general heterogeneous reasoning project. One is the shared view that there has until quite recently been a long-standing prejudice against non-symbolic representation in logic, mathematics, and computer science. The other is the assumed distinction between symbolic and diagrammatic systems. I do
share the view that there is a prevailing bias against non-symbolic sys-

tems, and I do assume that symbols are different from diagrams. How-

ever, I believe that in order to achieve further fruitful results the time has

come to raise more fundamental questions about the main assumptions

behind this fast-growing body of research into heterogeneous systems.

In the present work, I would like to emphasize the following circular

relation between these two points—the strong preference for symbolic

over diagrammatic systems and the distinct difference between these two types

of systems. Without a solid theoretical background for the distinction

between symbolic and diagrammatic systems, we easily overlook different

kinds of strength and weaknesses that each type of system possesses.

Then, given the general predominance of symbolic systems in the study

of logic, we tend to try to understand and evaluate diagrammatic sys-

tems against the criteria of a symbolic system. As a result, without their

own strengths being discovered, diagrammatic systems have been criti-

cized mainly because they lack the properties of a symbolic system. In

turn, this unfortunate result—stemming partly from a prejudice against

diagrams and partly from an unclear distinction between symbols and
diagrams—only reinforces the existing prejudice against non-symbolic

systems. I claim that these two main phenomena reinforce each other in

a vicious circle.

Where do we need to step in to break this circle? First of all, we

should stop evaluating diagrammatic systems in terms of symbolic sys-
tems. This would be possible only if we could develop an independent

way of approaching diagrams. Again, an independent method cannot

become available without identifying the positive properties that dia-

grammatic systems uniquely possess.

In this book, I examine Charles S. Peirce’s “Existential Graphs” as a

case study to support my claim that a prejudice against diagrammatic

systems reinforced a superficial distinction between symbols and dia-

grams and vice versa, and to show how this vicious circle can be broken.

Let me briefly explain below why Peirce’s graphical system provides us

with an exceptionally fitting case study for our present inquiry.

At the dawn of modern logic, Charles S. Peirce invented two different

types of logical systems, one symbolic and the other graphical. Why has

the symbolic system absolutely dominated the other in the subsequent
development of logic, despite the fact that Peirce himself considered EG superior to his symbolic logical system? We should be careful not to answer this question by citing the existing prejudice against diagrammatic systems. That would not explain much about the nature of the prejudice we are interested in investigating. More importantly, complaints against EG are different from complaints against the use of diagrams in proofs. Diagrams, critics say, are not rigorous enough to be used in a proof or tend to mislead us in a proof. Accordingly, diagrams are used with caution and only as heuristic aids. However, in the case of EG, logicians accept that EG is sound and complete. In spite of its rigor, EG has not been adopted for deductive reasoning. In this case, the criticism is more specific: The graphs of EG are difficult to read off, and the rules of EG difficult to use.

Why does a translation of a graph of EG almost always result in a complicated-looking sentence? Why is EG harder to use than natural deductive systems in spite of the apparent similarity between how these two systems are set up? Surprisingly, I find the same source responsible for these two puzzles: Despite the fact that EG is known to be a graphical system, the iconic features of EG have not been fully implemented either in any existing reading algorithm or in how the transformation rules are stated. This deficiency has yielded complicated and unintuitive reading methods and produced transformation rules that are not fine-grained enough to be used conveniently. A somewhat ironic, trivial-sounding but unobserved lesson follows: When a system is heterogeneous or diagrammatic, its iconic features should be fully utilized both in its syntax and semantics. Otherwise, many kinds of inefficiency will arise. Therefore, the existing criticisms against EG are the necessary product of the traditional, less then fully iconic way of approaching EG, and I argue that my new ways of understanding EG disarm the main criticisms of EG.

What, then, is the main difference between the traditional method and the new method I will present in this book? I will show that a strong dominance of symbolic over diagrammatic systems is present in how we traditionally have understood EG: Graphs of EG are read off very much as symbolic sentences are, and the inference rules of EG are understood as being like the rules of natural deductive systems. This
misguided assimilation of EG into a symbolic system, I claim, is the most fundamental source for the existing complaints against EG, which, in turn, supports a bias against non-symbolic systems. Here we confirm the existence of the vicious circle pattern. In order to correct this situation, I will rely on my discovery of the fundamental differences between EG and a logically equivalent symbolic system in two major respects.

First, I demonstrate how differently a meaningful unit of each system can be read off. In order to prevent ambiguity, the semantic interpretation of a symbolic sentence requires its unique readability, and hence no possibility of multiple readings. On the other hand, in the case of EG, multiple readings do not generate ambiguity. Moreover, multiple readings for one and the same graph are the most natural since we perceive a graph differently depending on how we carve it up. Importantly, we may obtain a comprehensive algorithm of multiple readings when we utilize more fine-grained visual features present in graphs. Second, I claim that the naturalness of the inference rules of a system is a relative concept. In the case of a natural deductive system, inference rules are stated so as to reflect the derivational history of a formula. However, in the case of EG, since so many different derivations are available for a graph, we should free the transformation rules of EG from any derivational history of a graph and look for a type of naturalness appropriate to a graphical system.

To find this type of naturalness both in the reading of graphs and in how inference rules are set up, the current work presents a new reading method of graphs and a newly formulated system of EG. Hence, we may meet the traditional objections to EG—that the system is hard to read off and hard to manipulate. Furthermore, this case study offers a new approach to non-symbolic systems in general. It urges heterogenous-system researchers to ask whether they fully utilize visual aspects of a non-symbolic system for a direct and natural reading algorithm, for an efficient formulation of inference rules, and for an intuitive interpretation of these rules. If we do not take advantage of visual distinctions already present in a system, we can only expect a graphical system to be less useful or less intuitive than the symbolic system whose criteria have been used to interpret the graphical system. Unless we adopt independent methods or criteria for a graphical system, we cannot
challenge the long-standing prejudice against non-symbolic systems, but only reinforce it.

This project draws on results from various disciplines concerned with diagrams. A distinct role that diagrams or pictures—as opposed to traditional linguistic forms—play in our cognition has been extensively discussed in several different areas, including philosophy, cognitive science, logic, artificial intelligence, and design theory. After outlining how various disciplines pursue this topic from slightly different points of view and how the relation between diagrams and representation (or reasoning) has been emerging as an interdisciplinary topic out of these different approaches, I explain how the project of this book fits in this overall picture.

Among many different approaches to the relation between diagrams (or pictures) and our cognition, I would like to draw attention to an interesting and useful distinction between those approaches that focus on diagrams as internal representations and those that treat them as external representations. In the Introduction to one of the most comprehensive anthologies on this subject, Chandrasekaran, Glasgow, and Narayanan (1995) make the following distinction between internal versus external diagrammatic representations:

- **External diagrammatic representations** These are constructed by the agent in a medium in the external world (paper, etc), but are meant as representations by the agent.

- **Internal diagrams or images** These comprise the (controversial) internal representations that are posited to have some pictorial properties. From now on, unless I specify otherwise, I will use the words ‘diagrams’ or ‘pictures’ to refer to external representations, and ‘images’ to refer to internal representations. Of course, many do not doubt that these two different levels of representation are closely related to each other. Moreover, this distinction is not needed at all for certain projects, and hence some researchers conflate internal and external representations. However, this distinction will nicely serve as a useful framework which I would like to use to show how different areas falling under the category of research on diagrammatic reasoning are related to one another.
The imagery debate between pictorialists and descriptionalists, one of the most time-honored controversies in psychology, focuses on our internal mental representation: It is about whether picture-like images exist as mental representations. Kosslyn and other pictorialists present a series of experimental data to support their position that some of our mental images are more like pictures than a linear form of language (for example, natural languages or artificial symbolic languages) in some important aspects, even though not all visual mental images and pictures are exactly of the same kind. By contrast, Pylyshyn and other descriptionalists raise questions about the status of picture-like mental images and argue that mental images are formed out of structured descriptions. To them, mental images represent in the manner of language rather than pictures, and hence no picture-like images play an important role in our cognition.

At this point we are not far from philosophical territory—the philosophy of mind. Philosophers’ deep interest in mental representation easily goes back to ancient times. Nobody would be surprised to realize that mental images were heatedly discussed during the heyday of ideas. As we know, Hobbes’, Locke’s, Berkeley’s, and Hume’s writings concern themselves in large part with mental discourse, the meaning of words, mental images, particular ideas, abstract ideas, impressions, etc. Descartes’ well-known distinction between imagining and conceiving something has generated much discussion about the unique role of visual images in mental representations. In the twentieth century, pictorialists in the imagery debate found the modern sense-datum theory in philosophy quite close to their point of view. By the same token, the critics of the sense-datum theory argued that the mistaken pictorial view of mental images arises mainly from our confusion about ordinary language. Not surprisingly, they are sympathetic to the view that mental images are epiphenomena. Contemporary philosophers, mainly in the philosophy of mind, have participated in a recent imagery debate among cognitive scientists.

Being slightly distant from the imagery debate itself, some cognitive scientists have concentrated on the functions of mental images or diagrams in our various cognitive activities, for example, memory, imagination, perception, navigation, inference, problem-solving, etc.
instead of exploring the ontological status of internal visual images.\textsuperscript{18} Here the distinct nature of “visual information,” which is obtained either through internal mental images or through externally drawn diagrams, has become a major topic of research. In particular, research on heterogeneous reasoning focuses on how visual information plays a unique role in our reasoning process. In the following, let me outline three important aspects of this research, all of which are crucial to understand the background of my project.

First, when the distinct role of visual information in inference or reasoning becomes a main question, we find an interesting shift of focus among researchers from internal to external representations, while we do not find a similar tendency in other subareas of research on images and cognition. For example, the study on the relation between memory and visual information focuses on internal as well as external visual information. However, in studies of inference, ‘images’ or ‘diagrams’ mainly refers to external representations, i.e., drawn pictures, graphs or diagrams, rather than picture-like mental representations.\textsuperscript{19} In Larkin and Simon’s classic paper “Why a diagram is (sometimes) worth ten thousand words” (1987), this change was made very clear: “Although our discussion [about the unique nature of diagrams in problem solving] may be relevant to this current controversy about the distinguishability of different internal representations, our analysis explicitly concerns external representations.”\textsuperscript{20} One benefit from this shift is that although consensus is lacking on whether there are different kinds of internal mental representations (as the long history of the imagery debate shows), everyone agrees that there are different forms of external representations.\textsuperscript{21}

Second, research on heterogenous reasoning has attracted much more attention from various different disciplines than has research on the relation between imagery and other cognitive activities—for example, memory or perception—mainly for two reasons. One is that human reasoning is a common topic among cognitive science, logic, mathematics, and artificial intelligence. The other reason for interdisciplinary interest in imagery and reasoning is that externally drawn diagrams or graphs, on which researchers have focused can, be a subject of all of these disciplines, unlike mental images.
Third, research on multi-modal representation has led researchers to explore the differences among different forms of (external) representations, but mainly between diagrammatic and symbolic representations. A strong dominance of symbolic languages in the study of representation systems (since the dawn of modern logic) compels anyone who is working on the relation between diagrams and inference to compare two different kinds of languages, that is, graphical and symbolic systems. In spite of a common interest in heterogeneous representation, various disciplines have pursued the same topics—here the relation between diagrams and reasoning, and the comparison between symbolic and graphical systems—from different points of view. After briefly summarizing the slightly different agendas of various disciplines, I will show how the work presented in this book serves to bridge the gap among them.

It has been a while since cognitive scientists started paying attention to how different forms of representation vary in their cognitive effects on human inference. Many important results have been produced along these lines. Based on Simon’s distinction between informational and computational equivalence among representations, Larkin and Simon (1987) present a case study in which two informationally equivalent systems, one sentential and the other diagrammatic, are shown to be computationally non-equivalent. Lindsay (1988) makes a related point by specifying where this computational difference lies. Claiming that an important role of diagrammatic representation in inference is not its expressive power, but its efficiency, he showed that this efficiency is obtained through the special properties that diagrams possess. Constraints built into the diagram-construction processes rule out many trivial cases, and, after the construction is completed, conclusions are directly read off from a diagram. Shimojima (1996) uses term ‘free ride’ to refer to an inference in which the conclusion seems to be read off almost automatically from the representation of premises. Gurr, Lee, and Stenning (1998) argue that the semantics of a diagrammatic system is more “direct” than the semantics of a symbolic system and that this crucial difference explains their characteristic low cost of reading off a conclusion. They also correctly point out that directness is relative, and hence, some rides are cheaper than others. Having a distinct role of
graphs in mind, Wang and Lee (1993) present a formal framework as a
guideline for correct visual languages. This impressive work provides
design theorists and AI researchers with computational support for
visual reasoning.

Not surprisingly, AI researchers, one of whose main concerns is the
heuristic power of a representation system in addition to its expressive
power, have been debating for decades about different forms of represen-
tation.26 Hence, they have welcomed discussions of the distinct role of
visual reasoning and have recently hosted interdisciplinary symposiums
on diagrammatic reasoning at AI conferences.27 At the same time, real-
izing that human beings adopt different representation forms depending
on the kinds of problems they face, some AI researchers and design the-
orists have practiced domain-specific approaches to bringing in problem-
tailored representation forms.28 Harel’s invention of higraphs (1988) is
an excellent example of obtaining practical results without being bogged
down in a more abstract and theoretical controversy. However, it is
likely that both top-down and bottom-up approaches to the distinct role
of diagrams in reasoning are necessary for the project to accomplish its
goal.

Heterogeneous reasoning has also been taken up by logicians. It is
important to note that logicians bring slightly different concerns to this
project from those of cognitive scientists or of AI researchers. First,
logicians’ main interest is exclusively in externally drawn representation
systems, as opposed to internal mental representations. Second, differ-
ences in cognitive effects or in heuristic power among different forms of
representation are not at the top of logicians’ agenda. The first and nec-
essary test for any representation system is to prove that the system is
correct. The next important question is the expressive power of the sys-
tem. If a language fails to justify its logical status or if its expressive
power is too limited, logicians’ interest in that language will fade. Ac-
cordingly, facing a strong prejudice against diagrams, which is more
deeply rooted in the modern history of logic than in any other area,
some logicians put their priority in examining whether there is any in-
trinsic reason why symbolic systems, but not diagrams, could provide us
with a rigorous proof. I took up this question with Venn diagrams and
showed that this system is not only sound and complete, but a slightly
modified version of the Venn system is logically equivalent to a monadic language.\textsuperscript{29} Hammer and Shin (1998) modified Euler diagrams to show that their modified version can be used in rigorous proof as well. Earlier works on Peirce’s Existential Graphs\textsuperscript{30} (in which graphical reasoning itself was not the main topic) can be re-evaluated to find significant contributions to the understanding of heterogeneous reasoning from logicians’ point of view. Based on existing case studies, Barwise and Etchemendy, the first two logicians who launched the inquiry into diagrammatic proofs in logic, conclude that “there is no principled distinction between inference formalisms that use text and those that use diagrams. One can have rigorous, logically sound (and complete) formal systems based on diagrams.”\textsuperscript{31} This conviction was necessary for the birth of their innovative computer program Hyperproof, which adopts both first-order languages and diagrams (in a multi-modal system) to teach elementary logic courses.\textsuperscript{32}

Each of these various research agendas is important to understanding heterogenous reasoning and needs to be taken seriously. With many concrete results in diagrammatic reasoning in hand, now is a good time to bridge the gap among the different strands of the research, which is one of the main goals of this work. First of all, EG is a sound and complete diagrammatic system, which makes logicians’ main worry about its logical status disappear. And EG is logically equivalent to a first-order language, which meets the criticism among logicians and mathematicians that a diagrammatic system cannot express as much as a symbolic system does. Also, unlike with domain-specific diagrams or pictures,\textsuperscript{33} this system is comprehensive and general enough to have the wider application required by AI researchers and design theorists.\textsuperscript{34} Moreover, this book will show how to make the existing system more efficacious and more natural by a new way of understanding graphs on their own terms, which is directly related to AI researchers’ and design theorists’ agendas. At the same time, abundant comparisons between EG and a logically equivalent symbolic language\textsuperscript{35} provide useful material to satisfy cognitive scientists’ main interests—cognitive differences between symbolic and diagrammatic systems.\textsuperscript{36}

The study of Peirce’s EG could not only bridge a gap among multiple agendas in contemporary research on multi-modal representation, but also provide us with a chance to explore a rather forgotten historical
fact that a diagrammatic representation system was invented at almost
the same time as a modern symbolic system was. Moreover, the first
comprehensive non-symbolic system, EG, was devised by Peirce, who is
one of the founders of modern symbolic logic. This interesting historical
fact encourages us to inquire into the philosophical motivation behind
Peirce’s invention of a graphical system.

Chapter 2 will reconstruct Peirce’s view of different logical notations
and different representations systems, which I claim to be the first com-
prehensive theory of heterogeneous reasoning. The evaluation of Peirce’s
work as a theory of heterogeneous reasoning is bound to be new for at
least two reasons. First of all, while many important works are available
about Peirce’s theory of signs and logical notation,37 the topic of hetero-
geogeneous reasoning has not been explored in the context of logical
theory, of formalization, or of the philosophical motivation behind EG.
More importantly, the area in which I claim Peirce’s contribution should
be recognized, i.e., heterogeneous reasoning, is a new interdisciplinary
research area, as explained above. I will show in chapter 3 that Peirce’s
view—the advocacy of a system with more than one kind of sign—is
implemented in his own graphical system. That is, chapter 3 presents the
thesis that EG is a heterogeneous representation system, by examining
both symbolic and iconic elements of the system.

Chapters 4 and 5 turn to more specific aspects of EG. Granting the
legitimacy of logicians’ criticisms against EG, in these two chapters I
present new methods of understanding the Alpha and Beta systems, re-
spectively. We can meet existing complaints against EG by discovering
more visual features present in graphs and by identifying the unique na-
ture of graphs. The fundamental differences between symbolic and dia-
grammatic systems are discussed in detail in these two chapters.

Chapter 6 raises a question as to the relation between Peirce’s theory
and his own practice: How did Peirce himself, who laid out the philo-
sophical groundwork for heterogeneous systems as shown in chapters 2
and 3, fail to obtain better ways of understanding his own graphical
system?38 For this curious historical question, I explore assumptions
behind Peirce’s distinction between logical systems and calculi and his
intention to make EG a logical system, not a calculus. Chapter 7 con-
cludes with a short summary of the book and suggestions for future re-
search on heterogeneous reasoning.
The history of modern logic has been one of intense investigation of symbolic systems. Interestingly, however, Charles Sanders Peirce, a founder of modern logic, developed two equivalent logical systems, one symbolic but the other graphical. The question I raise and aim to answer in this chapter is “Why did one of the founders of modern symbolic logic invent an elaborate diagrammatic representation system?” As is well known, the notations Frege adopted for his formalized language are more iconic than Peirce’s corresponding symbolic language. However, it was Peirce who presented a graphical system equivalent to a quantified language. I will argue that Peirce’s invention of a different kind of representation system is not just an accidental product of a logician’s mind, but a clear reflection of his philosophy of logic, which differed from that of contemporary logicians. In this chapter, I explore the philosophical roots of the birth of Peirce’s EG in his theory of representation and logical notation, which is the heart of Peirce’s philosophy of logic.

The first section introduces two insightful explanations related to our inquiry and concludes that more detailed explanations are needed to track down the philosophical motivation behind EG. The second section examines Peirce’s well-known phrase ‘diagrammatic reasoning’. This leads us to the investigation of Peirce’s theory of signs in the third section. After extending my analysis to some of the controversial contemporary topics in research on multi-modal reasoning, I claim in the last section that Peirce is the first person who presented a comprehensive theoretical groundwork for a heterogeneous formal system.
2.1 Preliminaries

Hintikka’s and Dipert’s evaluations of Peirce’s contribution to the history of modern logic allowed us not only to appreciate Peirce’s unique position in the history of logic but also to gain a more coherent understanding of Peirce’s own philosophy of logic. The latter achievement, as I will show below, advances us toward the goal of the current chapter, that is, to explore Peirce’s philosophical motivation behind his graphical system. As I point out at the end of each subsection, while both Hintikka’s and Dipert’s interpretations are consistent with Peirce’s invention of a graphical representation system, neither of them provides us with a philosophically sufficient account of Peirce’s invention of EG.

2.1.1 Graphs and the model-theoretic tradition

As the title suggests, van Heijenoort’s article “Logic as calculus and logic as language” (1967) draws a distinction between Boole’s *calculus ratiocinator* and Frege’s *lingua characterica* and makes an interesting connection between the *universality* of Frege’s *lingua characterica* and Frege’s lack of metatheoretical concepts. According to van Heijenoort, while Boole’s universe for a logical calculus does not have any ontological significance, and hence easily allows changes, “[f]or Frege it cannot be a question of changing universes,” since Frege’s logic presupposes that we have only one universe, i.e., the universe. The domain of the language includes everything, and there is no way for us to step back from this language to express ourselves outside of it. “[N]othing can be, or has to be, said outside of the system.” Therefore, van Heijenoort argues that there is no room for metalogic in Frege’s concept of logic.

When Goldfarb examines the nature of quantifiers in modern logic, he makes the following observation about logicists’ interpretation of quantifiers, which is consistent with van Heijenoort’s emphasis on Frege’s ‘the universe’:

The range of the quantifiers—as we would say—are fixed in advance once and for all. The universe of discourse is always the universe, appropriately striated. … For Frege and Russell … [e]very logical formula has a fixed meaning; there is no question of reinterpreting any sign. … Similarly, for Frege and Russell (or Russell at least until he became influenced by the doctrines of Wittgenstein), logic is about something, namely, everything.
The inflexibility of the interpretation of quantifiers prevents us from taking logic as subject-neutral or from applying logic to any particular domain in which we are interested. This, as Goldfarb says, is very distant from our view of logic. Thus, he arrives at a very similar evaluation of Frege’s and Russell’s project as does van Heijenoort—that a metalogical framework is not possible in Frege’s and Russell’s logicism.  

To arrive at metamathematics from Russell’s approach we must add the “meta,” that is, the possibility of examining logical systems from an external standpoint.

At the same time, Goldfarb draws our attention to the other tradition in modern logic, which allows for the concept of ‘meta’:

During roughly the same period [as Frege and Russell were working] a completely different approach to logic was being explored by Schröder and his followers. This tradition of the algebra of logic dates back to Boole’s work on the calculus of classes, a calculus also interpretable as a calculus of propositions. Building on earlier work of Peirce, . . . Schröder develops the calculus of relatives (that is, relations).  

The different approach Goldfarb describes here coincides with the tradition of a calculus ratiocinator that van Heijenoort discussed. The view of logic as a calculus permits the language to be reinterpreted so that we can apply it to particular domains or we can talk about the language from outside of it, in a metalanguage. This is why the concept of ‘meta’ can play a crucial role in the tradition of a calculus ratiocinator.

Based on a contrast between these two traditions, Hintikka draws a sharper distinction among modern logicians. Accepting van Heijenoort’s terminology, Hintikka calls Frege’s lingua characterica view the tradition of the universality of logic, but with an important modification. The universality of logic implies not only one universe, but only one language. Therefore, we are imprisoned both in this one world and in this one language. By contrast, Boole’s calculus ratiocinator view allows multiple domains and multiple interpretations, which is why Hintikka calls this position the model-theoretic tradition. Frege, Russell, Whitehead, Wittgenstein, and Quine are the main figures in the former, unitary language, tradition, and Boole, Schröder, Löwenheim, Gödel, the later Carnap, and Tarski the principals in the latter model-theoretic one.

When Hintikka makes important contrasts between Frege versus Peirce and places Peirce in the model-theoretic tradition, Peirce’s own
passages about EG are quoted as evidence for Hintikka’s claim—Peirce’s acceptance of Boole’s *calculus ratiocinator*:

Peirce himself identifies with perfect clarity the “very serious purpose” of his language of graphs, by saying that “this system is not intended to serve as a universal language for mathematicians or other reasoners like that of Peano” (4.424). . . . [T]his statement shows that Peirce was dealing with interpreted logic.¹³

Without discussing Peirce’s EG in any detail, Hintikka draws our attention to Peirce’s adoption of graphs or icons in a language of logic to support his main thesis that Peirce belongs to the model-theoretic tradition: “Hence Peirce’s willingness to theorize about icons and to use them in his actual work in logic is but another facet of his model-theoretical approach to language and logic.”¹⁴ The tradition to which Frege belongs does not allow us even to consider any other kind of language except the language we now have.

Hence, the following conclusion easily follows from Hintikka’s position: If Peirce had held the view of the universality of logic, he could not have invented a graphical system. Taking a logical language as a re-interpretable calculus was a necessary condition for Peirce to come up with a graphical system. This also explains why Frege, one of the two founders of modern logic, did not develop a graphical system, in spite of his iconic-looking notation for quantifiers. Frege’s view on universality left no room to consider any other logical system than the symbolic one that he had, and in particular no room for a non-symbolic language.

Thus, Hintikka’s evaluation of Peirce’s location in the history of logic is consistent with the birth of EG from Peirce’s philosophy of logic. However, it does not provide us with sufficient explanation of the birth of EG, since the model-theoretic view of logic does not necessitate a logical system other than symbolic ones. For example, Peirce could have invented more than one symbolic system to show that there is no one single universal language for logicians. Therefore, the differences Hintikka highlights between the two different logical theories do not complete the puzzle we are interested in solving in this chapter—“Why a *graphical* system (rather than more than one symbolic system)?” and “Why *this* graphical system?”
2.1.2 Graphs as a formal system

When Dipert examines Peirce’s place in the history of logic, he makes
an interesting observation that the meaning of ‘formal’ has shifted in a
wrong direction from the nineteenth to the twentieth century. While the
formal logic of the nineteenth century is not limited only to symbolic
logic, the more recent development of logic concentrates on symbolic
aspects of logic only:

We should also give some hard thought to the difficult question of how much
conceptual progress is made by symbolization and symbolic rigor alone.
Although Frege and Peirce are largely beyond suspicion, it is rather clear that
the recent history of logic has appeared to value any, and sometimes quite
shallow and unenlightening, symbolisms and axiomatizations and tended to dis-
miss any non-symbolic, historical account (for example, those of Aristotle or
Ockham) as so much empty verbiage.15

While he correctly criticizes the predominance of symbolization in the
development of twentieth-century formal logic, Dipert does not give any
substantial account of what non-symbolization is in formal logic. From
the following passage, it is rather clear that for Dipert two concepts—
the formal and the symbolic—are distinct from each other: “At root is a
twentieth-century misconception of what it is for a logic to be ‘formal’.
Namely, it need not be symbolic; nor is symbolization a guarantee of a
helpful formal analysis.”16

Then, what is non-symbolic formalism? In the previous quotation, his
phrase ‘any non-symbolic, historical account’ seems to suggest that a
historical account is one of the items that modern logic sacrifices for the
sake of symbolization. It is true that a historical account, such as Aris-
totle’s or Ockham’s, is not symbolic, but it is not formal either. So this
cannot be an example for non-symbolic formalism, but is rather non-
symbolic non-formalism. Even when Dipert gives credit to nineteenth-
century logicians for their correct use of ‘formal’ logic in the following
quotation, it is not clear what it means to be ‘formal’:

The nineteenth-century logicians, beginning with De Morgan’s Formal Logic,
kept in mind better than we what it is to be usefully ‘formal’ (that is, in attend-
ing to logical form) rather than merely symbolic at all costs.17

The phrase ‘usefully formal’ is ambiguous, since it could mean either
formal but not exclusively symbolic or not completely formal but with a
certain appropriate degree of formalism.18
Dipert correctly emphasizes that in the recent history of logic, symbolization has been almost the only trademark for formal logic. More importantly, Dipert aimed to convince the reader that nineteenth-century logic was not tied to symbolization as much as twentieth-century logic is. However, when Dipert relates these important historical data to Peirce’s contribution to modern logic, all he says is that Peirce, like Frege, made a substantial contribution to modern logic through rigorous and useful symbolization, unlike some modern logicians who adopt symbolism for the sake of symbolism. Here Peirce’s merit seems to lie in using symbolism not in a shallow but in an enlightening manner, so that it becomes usefully formal.

While Peirce’s appropriate use of symbolism cannot fully explain the invention of a non-symbolic system, Dipert’s vague idea that symbolism is different from formalism, I argue, is directly related to a better understanding of the root of EG in Peirce’s philosophy of logic. To formalize our reasoning is consistent with, but is not limited to, symbolization. Hence, non-symbolic formalization should be possible. Peirce’s invention of EG, a non-symbolic formal system, is the best piece of evidence to show that Peirce did not identify formalization and symbolization. Without a distinction between formalization and symbolization, Peirce could not have come up with two different kinds of logical systems, one symbolic and the other graphical. Dipert praised Peirce for being “the first great pioneer of the substantive (symbolic) theory of the logic of relations.” He should also have praised Peirce for not confusing the formal and the symbolic, unlike many modern logicians, and as evidence we can point to EG.

The recognition of the difference between symbolization and formalization was a necessary condition for the birth of EG, but not, however, a sufficient one. We need to know more about how Peirce differentiates symbolization and formalization and Peirce’s view of what it is for a language of logic to be formal. Therefore, understanding Peirce’s concept of a formal language (whether symbolic or not) is crucial to revealing the philosophical foundation for the birth of EG. Peirce, rather than any other previous logician, can give us a better understanding of what it is for a logical notation to be formal, since he himself presented two different kinds of formal systems, one symbolic and the other non-symbolic.